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The Reverse Arrangement Test: A Simple Procedure for Detecting Trends in Equipment Reliability

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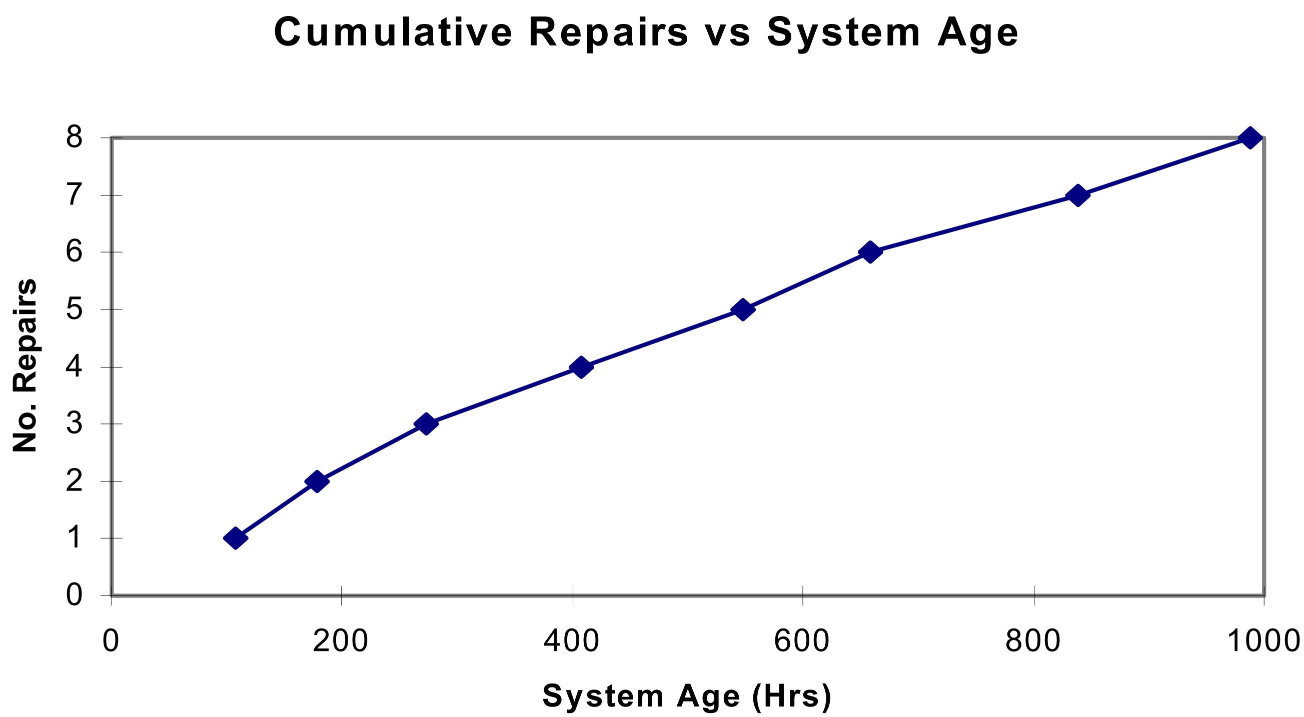
Objective and Background

- **Basic Problem:**
  - **Incorrect analysis of repairable system reliability data by applying techniques for the analysis of nonrepairable component data can lead to misleading conclusions**
- **Objectives:**
  - **To show simple graphical and analytical techniques for detecting trend in repairable system data**
- **Illustration:**
  - **Case study on a repairable system**

Case Study: System Repair History

Repairs were done at system times (hours) of 108, 178, 273, 408, 548, 658, 838, and 988

The cumulative repair plot is shown below.



Case Study Analysis

- **The repairable system was analyzed in traditional manner by taking times-between-repairs and treating data as a group of independent and identically distributed observations arising from a single population of failure times**

- **Methods for the analysis of nonrepairable components were used including Weibull probability plotting of data, parameter estimation, and model fitting**

Traditional Approach for Analysis of Data

- **The times between repairs (called the interarrival times) were calculated:**

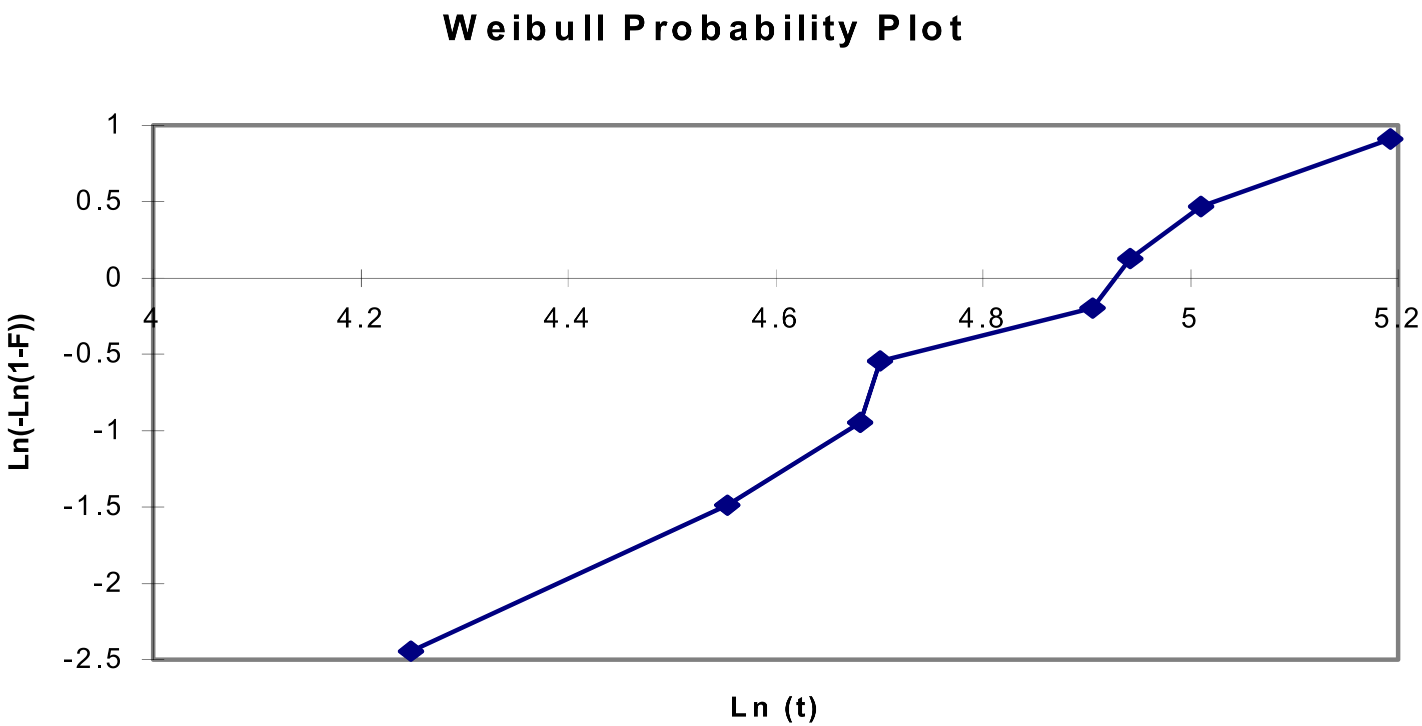
  108, 70, 95, 135, 140, 110, 180, and 150

- **The times were sorted and plotted on Weibull probability paper**

- **Since the plot showed reasonable fit to a straight line, the parameters of the Weibull distribution were estimated and used for assessing system performance**

Weibull Analysis of Repair Times

- **The sorted times are: 70, 95, 108, 110, 135, 140, 150, 180**
- **The associated plotting positions (median ranks) are (in percent): 8, 20, 32, 44, 56, 68, 80, 92**
- **The Weibull Probability plot is shown below.**

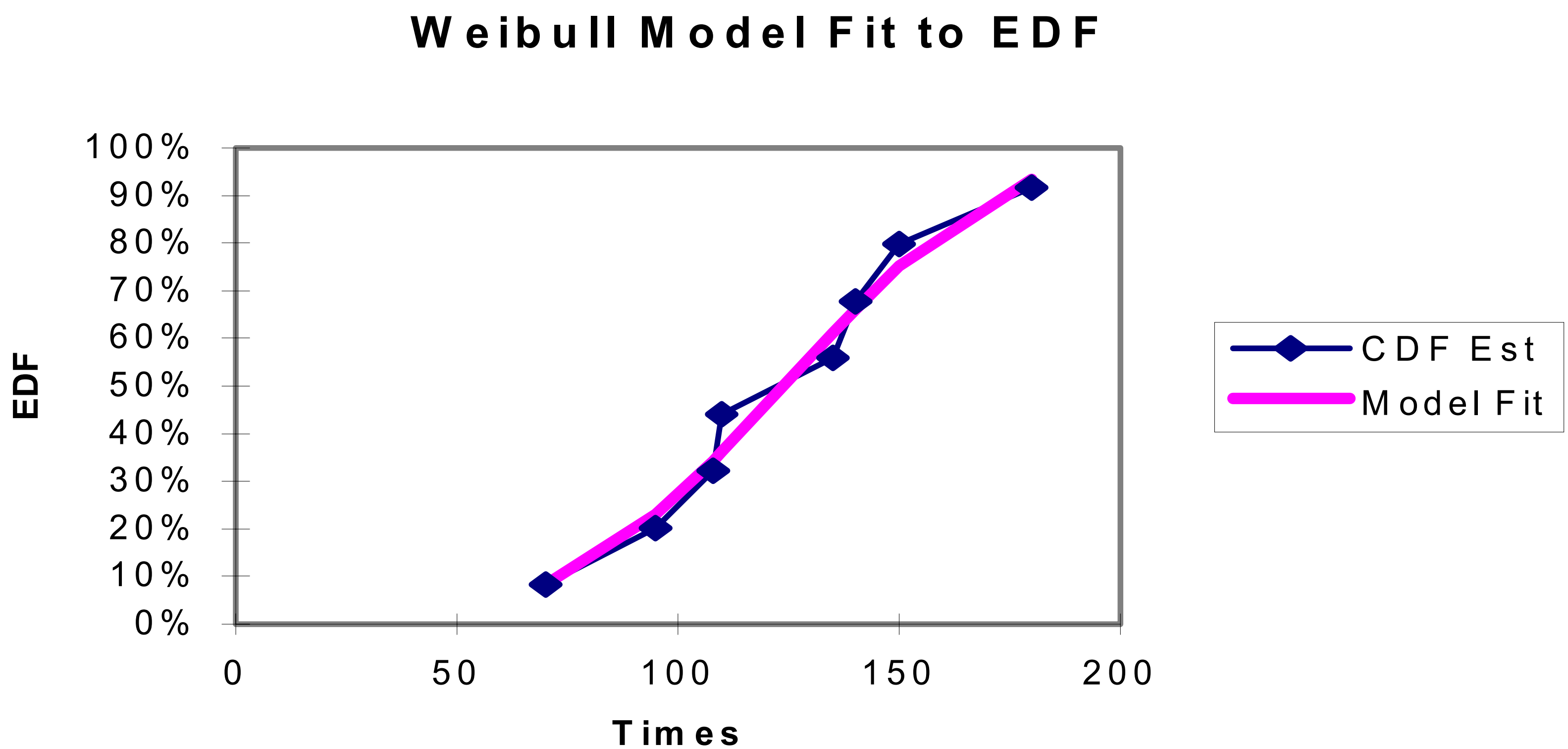


Weibull Parameter Estimation

- **A straight line appears to fit the data reasonably well.**
- **The shape parameter, *m*, is estimated by the slope of the line. Here the estimate is *m* = 3.65.**
- **The scale parameter, *c*, is estimated using the intercept in the equation: intercept = -*m* ln *c* . Here, the estimate is *c* = 137 hrs.**

Weibull Model Fit to Data

- **The graph below shows how well the Weibull model fits the empirical distribution function (EDF) where the data are treated as a single population of failure times**



Engineering Interpretation of Analysis

- **Analysis of data lead to conclusion that repair times followed a Weibull distribution**
- **The Weibull analysis indicated that the "failure rate" is increasing because the shape parameter, *m*, is greater than 1.0.**
- **Equipment engineers interpreted the results as indicating the machine needed to be brought down for repair and maintenance.**

A Review of Some Concepts for Analysis of Repairable Systems

- **A system is *repairable* if it can be restored to satisfactory operation by any action, including replacement of components, changes to adjustable settings, swapping of parts, or even a sharp blow with a hammer.**
- **Examples include:**
  - **TV's**
  - **Automobiles**
  - **Production equipment**

Reliability Issues for Repairable Systems

- **The failures occur sequentially in time**
- **Times between failures may not be independent and identically distributed (i.i.d.) observations from a single population, that is, a renewal process**
- **There may be stability, improvement, or degradation in the rate of repairs**
- **Order in which failures occur is important**
- **In nonrepairable component analysis, order of failures is ignored and times between failures are considered independent observations from a single population**

Repairable Systems Issues

- **Is there any trend indicating improvement or deterioration occurring?**
- **Are reliability objectives being met?**
- **What design or operation factors influence repair frequency?**
- **Are maintenance schedules appropriate?**
- **Are the provisions for spare parts adequate?**
- **Is there reliability growth?**

Value of Reliability Analysis

- To estimate burn-in requirements
- To provide for spare parts
- To forecast repair and warranty costs
- To upgrade existing systems
- To design better future systems
- To set-up maintenance schedules

Objectives for Analyzing Data

Two possible areas of interest

- **Single system available: analyzed to understand behavior for possible reliability improvement of existing or future systems**
- **Many copies of systems available: analyzed to estimate the repair rate of a population of systems, possibly for specifying burn-in effectiveness**

For this talk, we will concern ourselves with analysis of the single system

Times to Repair

- **Function of many factors**
  - **Basic system design**
  - **Operating conditions**
  - **Type of repairs**
  - **Quality of repairs**
  - **Materials used**
- **For a single component system, restoration to "like new," such as replacement of the failed component with one from same population, implies a renewal process (i.i.d.). However, even replacement with identical components is no guarantee of a renewal process!**
- **If inter-repair times are not i.i.d., renewal model is not valid and special techniques for analysis are required.**

Renewal Processes

- **Special case of repairable system**
- **Times between failures are i.i.d. from a single population**
- **No trend, stable repair rate**
- **Reliability analysis methods for non-repairable components have applicability**

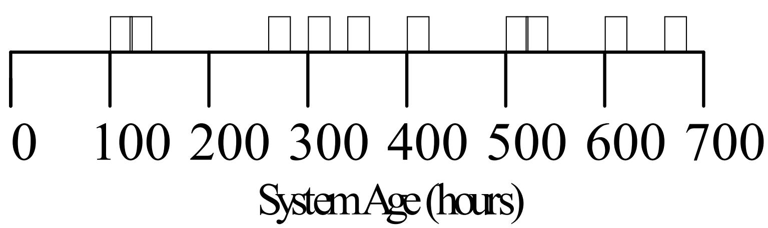
Analysis of Renewal Process

Consider a single system for which the times to make repairs are ignored

Ten failures are reported at the system ages (in hours):

106, 132, 289, 309, 352, 407, 523, 544, 611, 660.

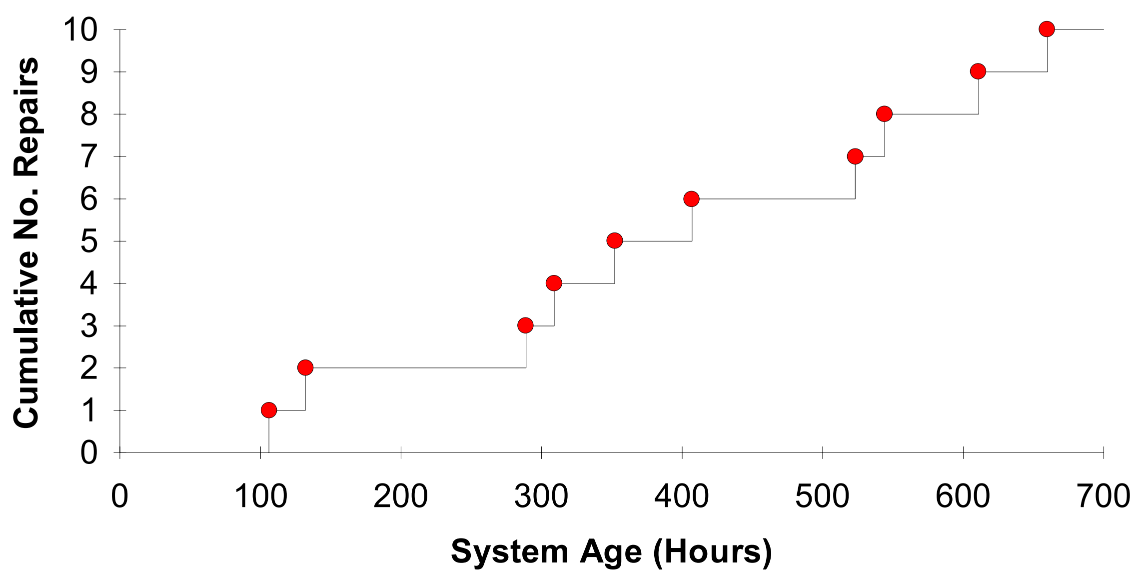
The pattern of repairs is



Cumulative Plot

- **The most common data graph is called the cumulative plot: the cumulative number of repairs is plotted against the system age.**
- **For the data shown, the cumulative plot is:**

Figure 10.1 Cumulative Plot



Analysis of a Renewal Process

Under a renewal process, the times between failures are i.i.d., that is, from a single population having a fixed mean (average) repair time.

Consequently, the cumulative plot should appear to follow a straight line.

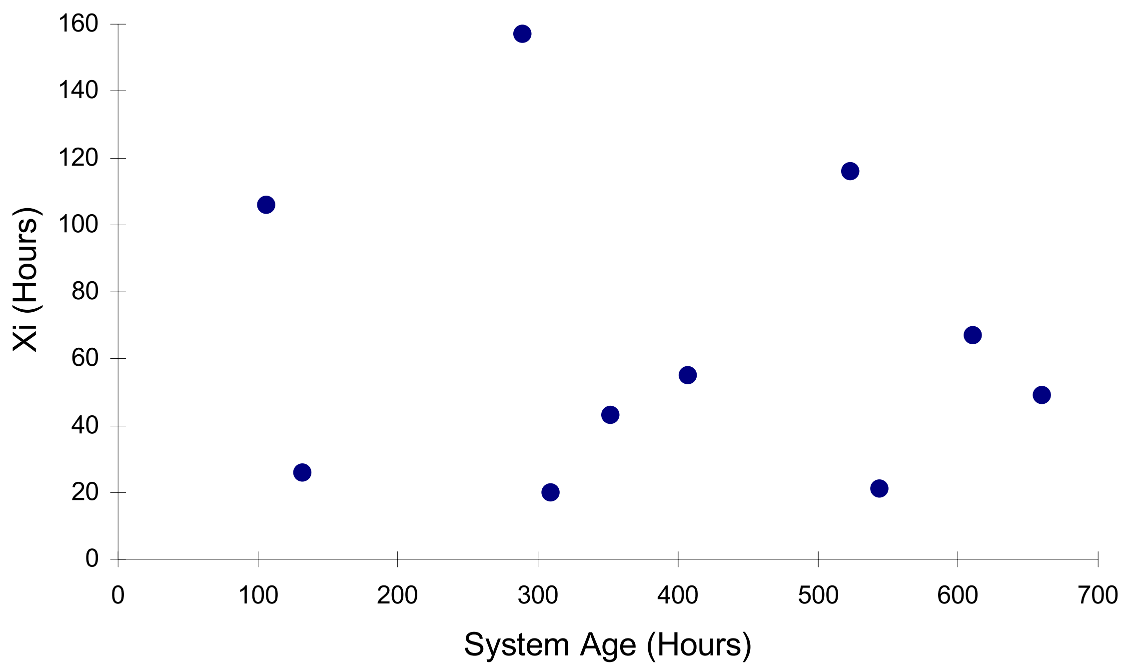
Interarrival Times

Look at the times between repairs, called the *interarrival times*:
106, 26, 157, 20, 43, 55, 116, 21, 67, 49.

A useful chart is a plot of the interarrival times versus the system age at repair.

Interarrival Times Versus System Age

A very useful chart.



Renewal Process Analysis

If a renewal process exists, we can treat the sample of ten (assumed) independent observations (that is, the interarrival times) as arising from a single population.

We can analyze the data using methods for non-repairable components. Thus, we can sort the data and plot on probability paper or use MLE methods.

Special Renewal Process: Homogeneous Poisson Process

Suppose the interarrival times X_i are i.i.d. with exponential pdf having failure rate λ, that is,

$$f(x) = \lambda e^{-\lambda x}$$

Then, we can show that the total number of repairs by time *t*, denoted by N(t), has a Poisson distribution with mean rate (constant intensity) λ

Thus, the probability of observing exactly $N(t) = k$ failures in the interval (0,*t*) is

$$P[N(t) = k] = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

Homogeneous Poisson Process

A renewal process in which the interarrival distribution is exponential is called a homogeneous Poisson process (HPP).

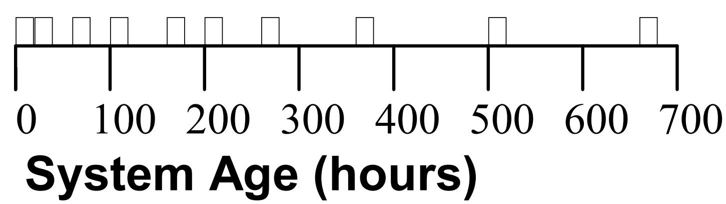
The expected value for *N*(*t*) is λt

The mean time to the kth repair is k /λ

Graphical Analysis of Non-Renewal Processes

Suppose the observed consecutive repairs times were 20, 41, 67, 110, 159, 214, 281, 503, 660.

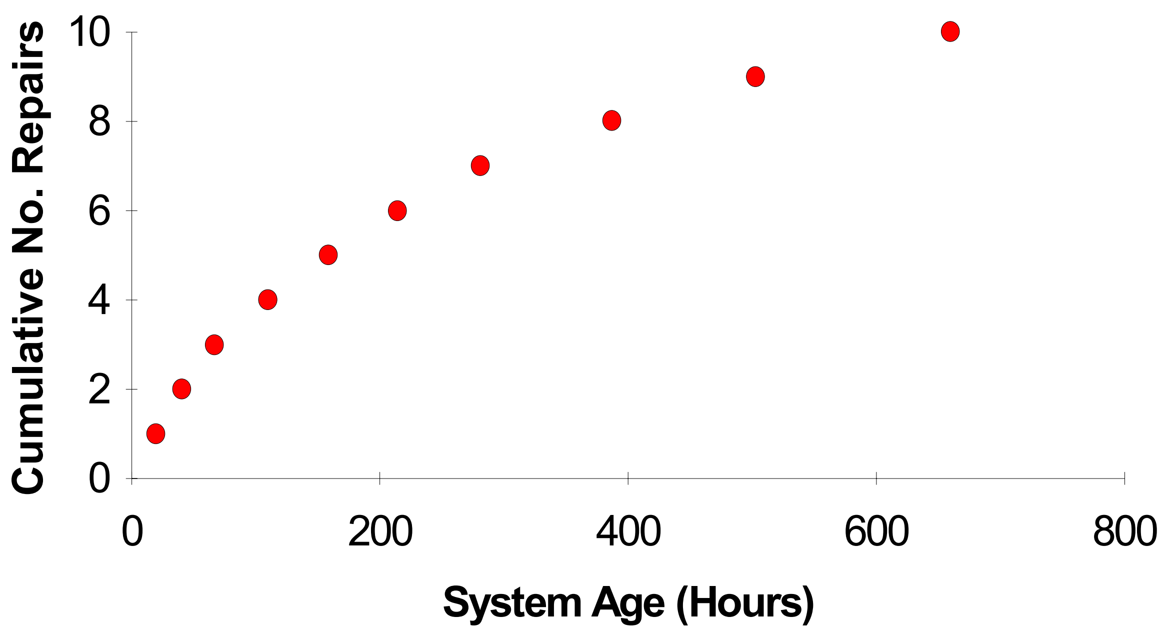
A line sketch of the pattern of repairs shows:



Cumulative Plot

The cumulative plot for this set of data is shown below.

Figure 10.4 Cumulative Plot - Improving



The curvature suggests a decreasing frequency of repairs, that is, an improving failure rate.

Interarrival Times

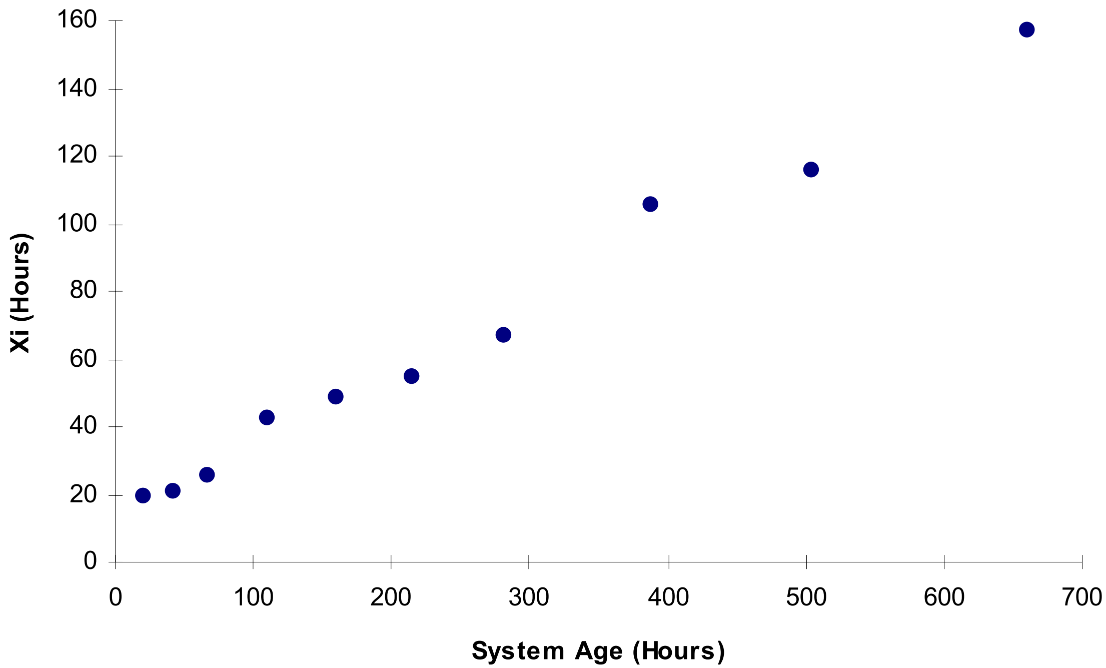
For this set of data, the interarrival times are
20, 21, 26, 43, 49, 55, 67, 106, 116, 157.

These are exactly the same interarrival times as those for the renewal process!

Now the order in which the interarrival times appear is important.

Can't use standard non-repairable methods, such a probability plotting, to analyze data.

Interarrival Times Versus System Age, Improving

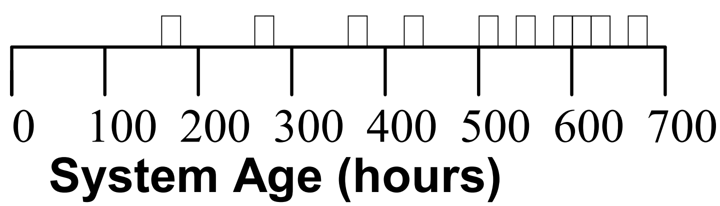


Another Repairable System History

Suppose the repairs occurred at the following times

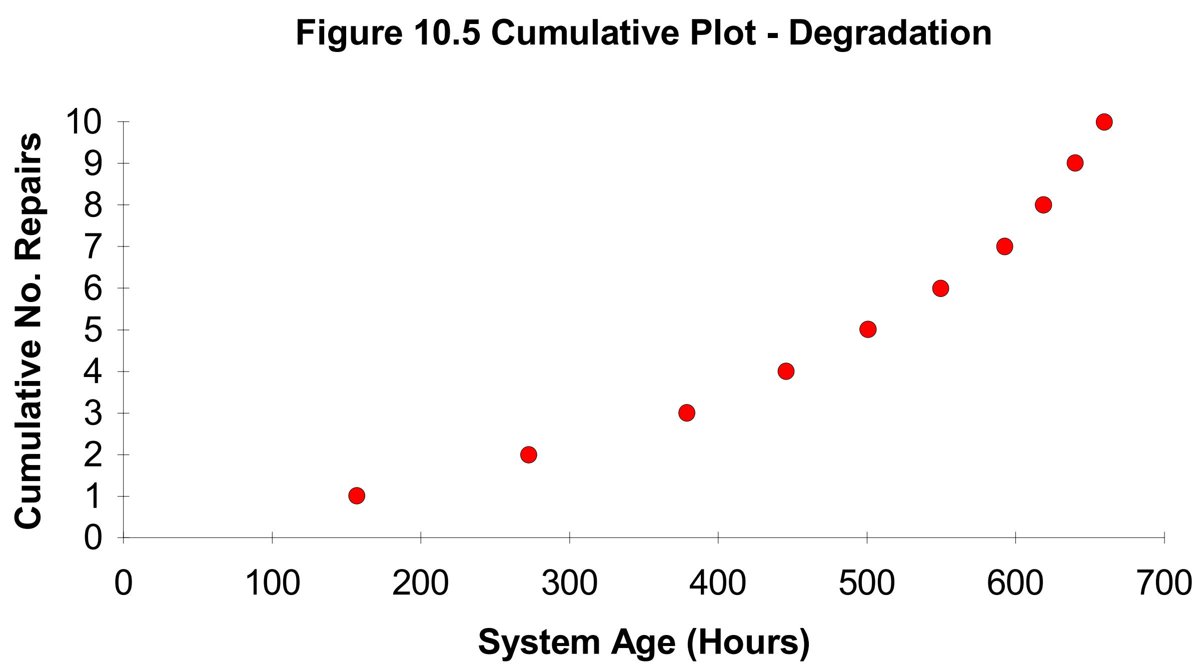
157, 273, 379,446, 501, 550, 593, 619, 640, 660.

The line sketch is



Cumulative Plot

The cumulative plot is shown below.



The curvature shows the frequency of repairs increasing in time, indicating system degradation.

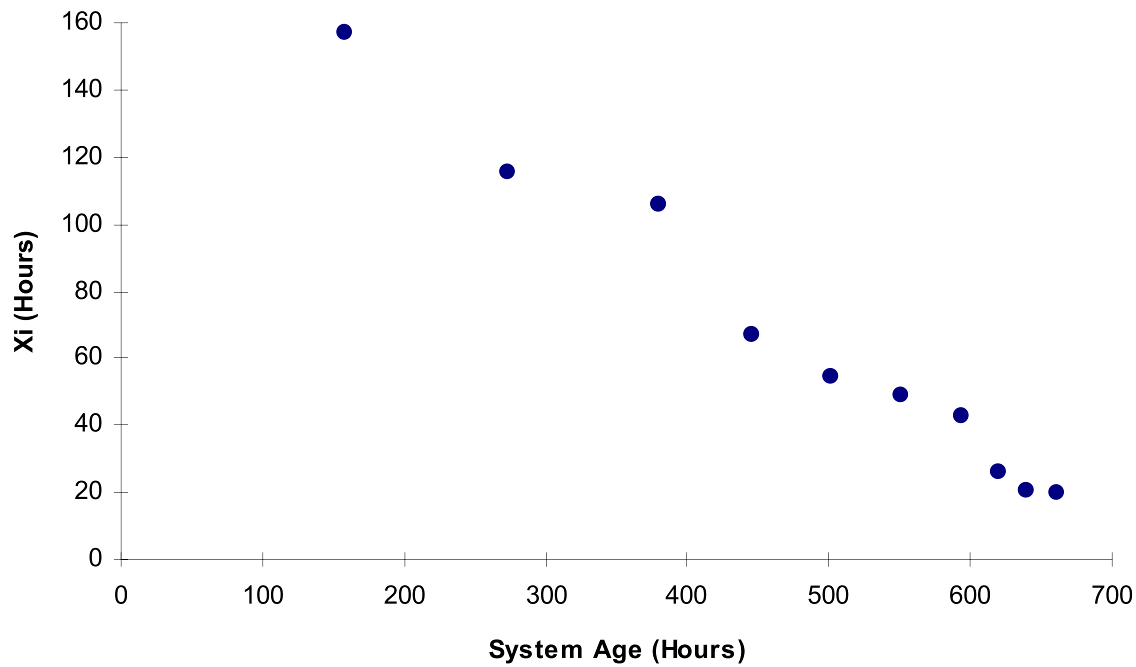
Interarrival Times

The interarrival times are
157, 116, 106, 67, 55, 49, 43, 26, 21, 20.

These are exactly the same times as the last two sets of data! Only the order is different.

Again, it is not correct to analyze the time between repairs as if they were independent observations from a single population.

Interarrival Times Versus System Age, Degrading



Testing for Trends and Randomness

Model assumptions should be verified.

Sequence of checks is as follows:

- **Plot the data.**
- **Check for trend**
  - » **yes: NHPP or other nonstationary models**
  - » **no: continue**
- **Check for identically distributed and independent**
  - » **i.d. & independent: renewal process (continue)**
  - » **i.d. & not independent: branching Poisson process or other model**
  - » **not i.i.d.: possibly subdivide data**
- **Check if exponential distribution for interarrival times**
  - » **yes: HPP**
  - » **no: other models or distribution free methods**

Analytical Tools to Check Trend

Laplace Test

- Tests whether or not an observed series of events is a HPP
- The test statistic is

$$L = \frac{\sum_{i=1}^{n} T_i - \frac{nT}{2}}{T\sqrt{n/12}}$$

which approaches a standard normal variate under HPP for moderately large *n*.
Can combine multiple systems into Laplace test statistic.

Reverse Arrangement Test (RAT)

Nonparametric Test (No Distribution Assumed)

Consider set of interarrival times occurring in the sequence

$$X_1,\ X_2,\ \ldots,\ X_n$$

Define a reversal as each instance of an earlier repair being smaller than subsequent times, that is,

$$X_i < X_j$$

for $i < j$, where $i = 1, \ldots, n-1$ and $j = 2, \ldots, n$

Counting Reversals Example

Example:

System ages at repairs: 25, 175, 250, and 350.
Interarrival times are 25, 150, 75, 100.

There are 3 reversals for the first time of repair 25, since that time is less than next three.
There are zero reversals for the second time 150 which is larger than the next two.
There is one reversal for the time 75 which is smaller than the last.

Thus, the series of interarrival times has 3+0+1=4 reversals.

RAT Criteria

Too many reversals indicate increasing trend; too few, consistent with decreasing trend.

Statistically, we can calculate, for *n* repair times, tables of critical values for a specific number of reversals (see *Applied Reliability*, 2nd ed.) to reject evidence of no trend in the data series and thereby conclude a trend does exist.

Determining RAT Critical Values

Consider *n* = 4 observations, designated

$$X_1, X_2, X_3, X_4$$

There are 4!=24 possible permutations. For example, here are a few:

$$X_1 X_2 X_3 X_4$$

$$X_1 X_2 X_4 X_3$$

$$X_1 X_3 X_2 X_4$$

$$X_1 X_3 X_4 X_2$$

$$X_1 X_4 X_2 X_3$$

$$X_1 X_4 X_3 X_2$$

$$X_2 X_1 X_3 X_4$$

$$X_2 X_1 X_4 X_3$$

and so on.

Maximum Number of Reversals

We can show that the maximum number of reversals for a series of *n* times is

$$n(n-1)/2.$$

So for n = 4, we have a maximum of

$$4(3)/2 = 6.$$

By counting the number of reversals for each permutation, we can calculate the probability of zero to 6 reversals occurring by chance.

Enumerating Reversals

For our *n* = 4 example, the sequence

$$X_1 < X_2 < X_3 < X_4$$

is the only permutation with six reversals.

There are only 3 permutations that give 1 reversal

$X_4 X_3 X_1 X_2$ $X_3 X_4 X_2 X_1$ $X_4 X_2 X_3 X_1$

and so the probability of exactly 1 reversal is 3/24.

Next, we can determine which permutations give 2 reversals and so on.

Example RAT Probabilities

For n = 4, the probability of 0, 1, 2, 3, 4, 5, 6 reversals is 1/24, 3/24, 5/24, 6/24, 5/24, 3/24, 1/24, respectively.

Since 1/24 = 4.2%, we see that 0 or 6 reversals is significant at the upper or lower 5% significance level.

We can create tables of critical reversal numbers for different n.

Table of Critical Values for RAT

Table 10.6 Critical Values $R_{n,\%}$ of the Number of Reversals for the Reverse Arrangement Test

Sample Size	*Single-Sided Lower Significance Level*			*Single-Sided Upper Significance Level*		
n	**1%**	**5%**	**10%**	**10%**	**5%**	**1%**
4		0	0	6	6	
5	0	1	1	9	9	10
6	1	2	3	12	13	14
7	2	4	5	16	17	19
8	4	6	8	20	22	24
9	6	9	11	25	27	30
10	9	12	14	31	33	36
11	12	16	18	37	39	43
12	16	20	23	43	46	50

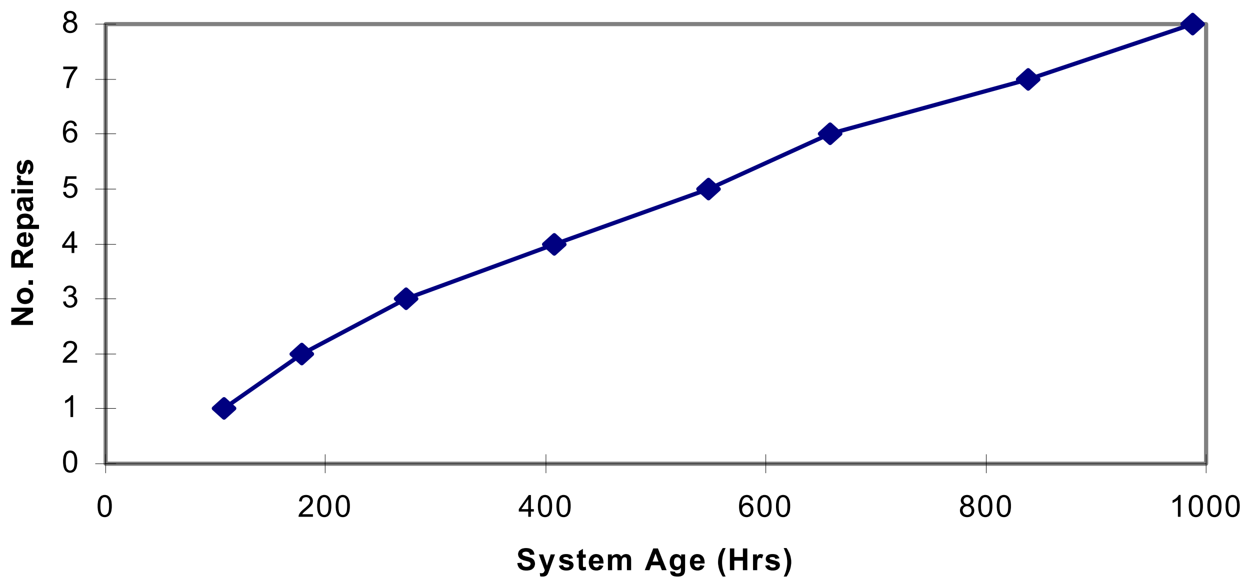
Too few reversals. (degradation)

Too many reversals. (improvement)

Case Study Example

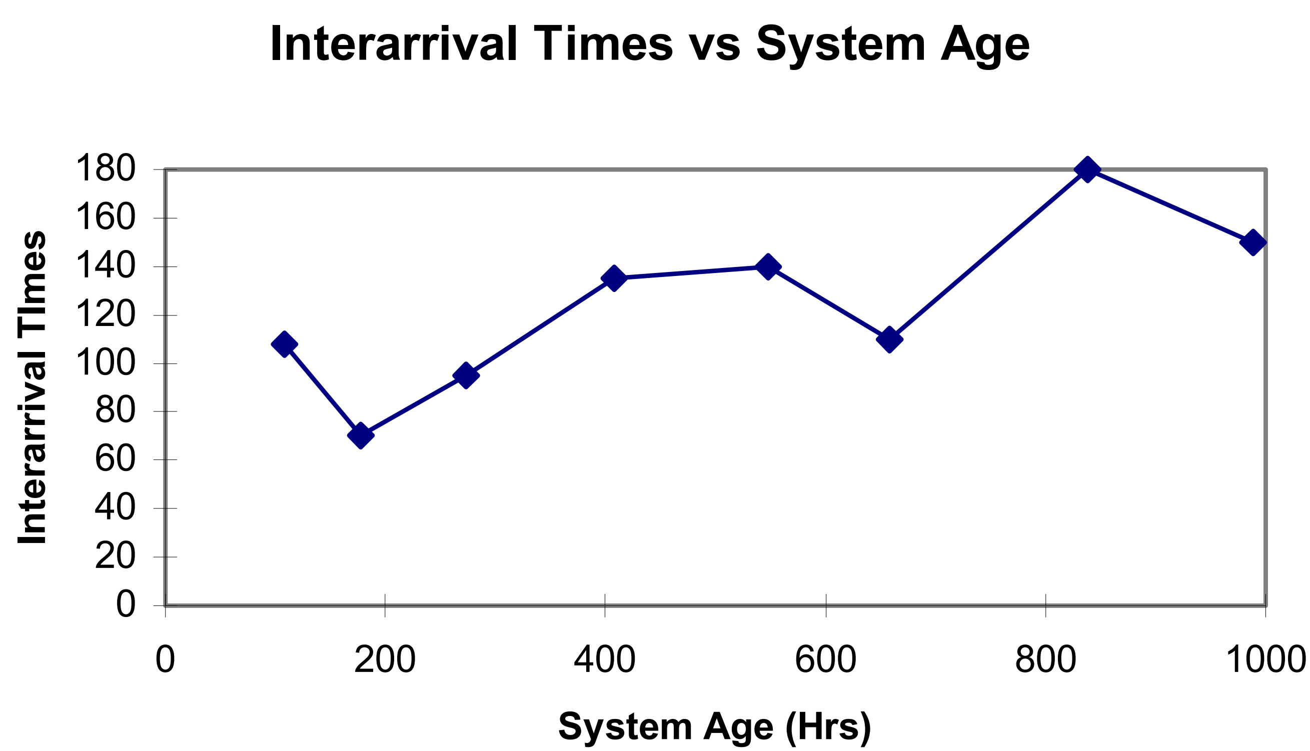
The system experienced repairs at the following ages
108, 178, 273, 408, 548, 658, 838, 988
Is there any evidence of a trend?

Cumulative Repairs vs System Age



Interarrival Times Versus System Age Plot

We plot the consecutive times between repairs versus the system age at repair.



Analytical Vs Graphical Analysis

There appears to be a trend visible in either plot.

How significant is it?

We need a statistical approach that tells us the likelihood of such a pattern if there really is no trend.

What is the probability of the observed sequence of repair times occurring by chance alone under a renewal process?

RATS Example

Solution:

The interarrival times are 108, 70, 95, 135, 140,110, 180, 150.

There are 5+6+5+3+2+2+0=23 reversals

Comparison to critical table shows 22 reversals in 8 items is significant at the 5% level. Hence, we reject any renewal process, and in particular, the HPP, as a suitable model.

With at least 95% confidence, we state that the system is improving in time.

Results/Implementation

- **The correct analysis showed an improving trend in the repairable system history**
- **Incorrect analysis lead to the belief that maintenance was necessary to restore reliability when such action might have made the reliability worse**
- **By using correct procedures to detect the trend, the realization of the improvement was made and corrective action halted**
- **Search for the source of the improvement was instead addressed leading to adoption of new techniques for repair**
- **The result was improved reliability for the existing system and the prospect of improved reliability for future systems**

Impact

- **By not performing unnecessary maintenance considerable savings in money and cycle time was possible**
- **Unnecessary repairs could have made the reliability worse**
- **If correct techniques are not employed, reliability improvement could be missed**
- **RAT is a simple test to apply**
- **Graphical procedure is effective**

Review

- **We have discussed various processes for repairable systems:**
  - **Renewal**
  - **Non-renewal**
- **We have presented both graphical and analytical methods for revealing trends.**
- **We have reviewed a simple procedure called RAT for performing a nonparametric test for trend in repairable system data.**

Summary

- **Important to verify assumptions in reliability analysis of repairable systems**
- **Analysis of repairable systems with techniques for non-repairable components can be misleading and costly**
- **Powerful graphical and analytical techniques exist for detecting trends in repairable system reliability**

Further Information

For further discussion of techniques for the analysis of data from repairable systems, see the text, *Applied Reliability, 2nd* edition, by Paul Tobias and Dave Trindade, published in 1995 by Van Nostrand Reinhold, New York, NY.

See also the paper by John Usher entitled "Case Study: Reliability Models and Misconceptions," in *Quality Engineering*, 6(2), pages 261-271 (1993-1994)