

## PREDICTION OF FAILURE RATE OF FIELD-EFFECT TRANSISTORS

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Failure rates of field-effect semiconductor devices may be predicted by the following steps: 1) determining the average percentage of a first sample of devices, which exhibit dielectric breakdown after a short period (e.g., 2 to 10 seconds) of current-limited voltage stress sufficient to cause normally good devices to go into saturation; 2) determining the average percentage of a second sample of devices, which exhibit dielectric breakdown after an extended period (i.e., over one-hundred hours) of low-voltage stress; 3) correlating the percentage of devices exhibiting dielectric breakdown under each stress condition, to determine the specific relationship between the percentages; 4) predicting the percentage of devices expected to exhibit breakdown at an extended period of time, by determining the average percentage of devices manufactured under slightly different manufacturing conditions which exhibit breakdown under the short current-limited voltage stress; and 5) applying the specific relationship previously determined.

## PROGRAM TO NUMERICALLY DIFFERENTIATE DATA

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In fields such as numerical analysis, statistics, reliability, or engineering, there is often the need to differentiate a set of data. For example, given X, Y values, it may be necessary to determine the minimum rate of change of Y versus X. A standard procedure would be to plot a graph of Y versus X, assume a function to represent the data, and use the derivative of the function to yield the rate of change of Y versus X. An alternative method, very simple in concept, would be to place a ruler by eye tangent to the empirical curve at various points and obtain an approximation to the derivative by the standard  $(\Delta X)/(\Delta Y)$  estimator. However, this process can be very tedious and time-consuming to do by hand, especially if many points are desired for the derivative. A program is described to numerically differentiate a curve by such a procedure.

Suppose the data consists of  $n$  (X,Y) pairs of data points. We choose the first  $p$  points on which to perform a least squares fit of Y versus X. In the least squares fit the sum of the squares of the vertical deviations of Y from the linear regression line is minimized. Thus, a regression line is fit to the first  $p$  data points, and the slope of the regression line is used as the estimator of the derivative. The point at which the derivative is considered evaluated is the average of the first  $p$  values of X, that is,  $\bar{X}_p$ . The program then steps up by one to the next  $p$  data points and performs a similar calculation. The process continues until the last  $p$  data points are fitted.

The output consists of  $n-p+1$  pairs of values (S, $\bar{X}$ ), where S is the value of the slope at the point  $\bar{X}$ . These points may be then graphically plotted to obtain a curve of the estimated derivative of the original curve.

For  $p = 2$ , the results will be the same as a point-by-point measure of the slope of the line connecting each data pair in the X, Y curve. Note that as the number  $p$  is increased, the number of points at which the slope is measured is decreased. Hence, the curve of the derivative versus  $\bar{X}$  becomes narrower (in  $\bar{X}$ ) as the ends move in. This feature occurs because more data points are used to estimate the slope at each point of  $\bar{X}$ . This alone results in a smoothing of the derivative of the data. Thus,  $p$  can be used to control the range of derivative values, that is, the degree of smoothing in Y over a progressive narrowing range of X. In practice, a value of  $p$  approximately equal to 20-25% of  $n$  has been found useful.

PROGRAM TO NUMERICALLY DIFFERENTIATE DATA - Continued

This program can be written in any language. However, APL is currently used in our area to determine failure rates by numerical differentiation of cumulative distribution function (CDF) curves.

## RENEWAL FUNCTION ESTIMATION

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This article sets forth a more efficient algorithm to estimate the renewal function under multi-censoring of systems.

In a group of  $N$  systems where each system consists of similar components, the failure of a component causes the systems to cease operation. However, the system can be immediately restored to operation by replacement of the failed component with another component from the same population. The systems may have different entry times into operation, resulting in different current operating times. Thus there is progressive or multiple censoring of groups of component lifetimes.

The renewal function  $M(t)$  is defined as the expected number of component renewals or replacements by time  $t$  at a single component position. By pooling the system failure data, the renewal function necessary for the evaluation of field reliability can be estimated.

An unbiased estimator of  $M(t)$ , for a single system of  $c$  components, is just

$$\hat{M}(t) = n(t)/c,$$

where  $n(t)$  is the number of renewals on all component positions by time  $t$ . Similarly, if  $Q$  systems have not experienced any censoring (losses) by time  $t$ , an unbiased estimator is  $M(t) = n(t)/cQ$ , where  $n(t)$  is now the total number of renewals on all systems by time  $t$ .

For multiple systems after one or more systems have been lost, however, a choice of methods for estimating  $M(t)$  arises. For example, suppose there are two systems each containing  $c$  components and  $t > T_1$ ,  $T_1$  being the loss time for system 1. A "reduced sample approach" would specify using the second system alone, after  $T_1$ ; that is,

$$\hat{M}(t) = n_2(t)/c, \quad t > T_1,$$

where  $n_2(t)$  is the number of renewals by time  $t$  on system 2. This estimate is unbiased.

An alternative is called the pooled summation approach, and this method uses both systems to estimate  $M(t)$  for  $t \leq T_1$ , and then adds the average number of renewals on system 2 for  $t > T_1$  to the estimate at  $T_1$ . Thus, for  $t > T_1$ ,

RENEWAL FUNCTION ESTIMATION - Continued

$$\hat{M}(t) = \hat{M}(T_1) + \hat{M}(T_1, t),$$

where  $\hat{M}(T_1)$  is the total number of renewals  $n(t_1)$  on both systems by time  $T_1$  divided by the total number of components,

$$\hat{M}(T_1) = n(T_1)/2c,$$

and  $\hat{M}(T_1, t)$  is the number of renewals  $n_2(T_1, t)$  occurring in the interval  $(T_1, t)$  on system 2 divided by the number of components on system 2,  $M(T_1, t) = n_2(T_1, t)/c$ . There are obvious extensions to handle unequal numbers of components on systems.

For example, let system 1, with  $c = 5$ , have loss time  $T_1 = 1000$  hours and renewals at 500, 750, and 900 hours. Let system 2, with  $c = 5$ , have a renewal at 1200 hours. The reduced sample approach gives  $M(t) = 0.1, 0.2, 0.3, 0.2$ , respectively, at the times 500, 750, 900 and 1200 hours, while the pooled summation approach yields  $M(t) = 0.1, 0.2, 0.3, 0.5$ , respectively.

Note that the reduced sample and pooled summation estimators are step functions with jumps at the fail times. The latter estimator is also unbiased, but in addition, it is non-decreasing in time, a property of the renewal function but not the reduced sample estimator. At each system loss time, the pooled summation estimator  $M(t)$  is anchored at that point and subsequent renewals are summed to the previously anchored value. In the reduced sample approach, the estimator  $M(t)$  at later times may easily decrease if the system lost had an above average number of renewals up to its loss time.