

An EXCEL Add-In for Comparing Two Exponential Distributions

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2000 Joint Statistical Meetings, Indianapolis

Reliability Analysis Problem

- Exponentially distributed times to failure
- Want to compare two
 - MTTFs (non-repairable components)
- or
- MTBFs (repairable systems)
- Typically censored data (Time or Count censoring)

Statement of Problem

- Two exponentially distributed data sets of count censored data
- Let T_i be the total time on test, $i = 1, 2$

$$T_i = \sum_{j=1}^{r_i} t_{i(j)} + (n_i - r_i) t_{i(r_i)}$$

- The mean time to failure (MTTF) estimate is

$$\hat{\theta}_i = T_i / r_i$$

Distributions of Statistics

- T_i has a distribution given by

$$2T_i / \theta_i \sim \chi_{(2r_i)}^2$$

- The ratio

$$\frac{2T_1 / 2r_1\theta_1}{2T_2 / 2r_2\theta_2} = \frac{\hat{\theta}_1\theta_2}{\hat{\theta}_2\theta_1}$$

has an F -distribution with $(2r_1, 2r_2)$ degrees of freedom

Procedure

- Set up two-sided $100\gamma\%$ confidence interval for the ratio θ_1 / θ_2

- Confidence limits are

$$\underline{\rho} = (\hat{\theta}_1 / \hat{\theta}_2) / F[(1 + \gamma) / 2; 2r_1; 2r_2]$$

$$\tilde{\rho} = (\hat{\theta}_1 / \hat{\theta}_2) \cdot F[(1 + \gamma) / 2; 2r_2; 2r_1]$$

- If interval **excludes** 1, then θ_1 and θ_2 differ statistically at the specified significance level

Example (Count Censored)

Two exponentially distributed data sets

$$\hat{\theta}_1 = 12,600$$

$$\hat{\theta}_2 = 8,800$$

$$r_1 = 16$$

$$r_2 = 22$$

Ratio ($\hat{\theta}_1 / \hat{\theta}_2$) 1.43

95% CI: LCL 0.76

UCL 2.80

Interval includes 1.

Not significantly different.

Time Censored Data

The ratio

$$\frac{(1 + 0.5/r_1) \theta_2 \hat{\theta}_1}{(1 + 0.5/r_2) \theta_1 \hat{\theta}_2}$$

has an F -distribution with $(2r_1+1, 2r_2+1)$ degrees of freedom.

Procedure

- Set up two-sided $100\gamma\%$ confidence interval for the ratio θ_1 / θ_2

- Confidence limits are

$$\tilde{\rho} = \frac{(\hat{\theta}_1 / \hat{\theta}_2)(1 + 0.5/r_1)}{(1 + 0.5/r_2)} / F\left[\frac{(1 + \gamma)/2; 2r_1 + 1; 2r_2 + 1}\right]$$

$$\tilde{\rho} = \frac{(\hat{\theta}_1 / \hat{\theta}_2)(1 + 0.5/r_1)}{(1 + 0.5/r_2)} \cdot F\left[\frac{(1 + \gamma)/2; 2r_2 + 1; 2r_1 + 1}\right]$$

- If interval **excludes** 1, then θ_1 and θ_2 differ statistically at the specified significance level

Example (Time Censored)

Two exponentially distributed data sets

$$\hat{\theta}_1 = 120$$

$$\hat{\theta}_2 = 55$$

$$r_1 = 10$$

$$r_2 = 15$$

Ratio ($\hat{\theta}_1 / \hat{\theta}_2$) 2.18

95% CI: LCL 1.03

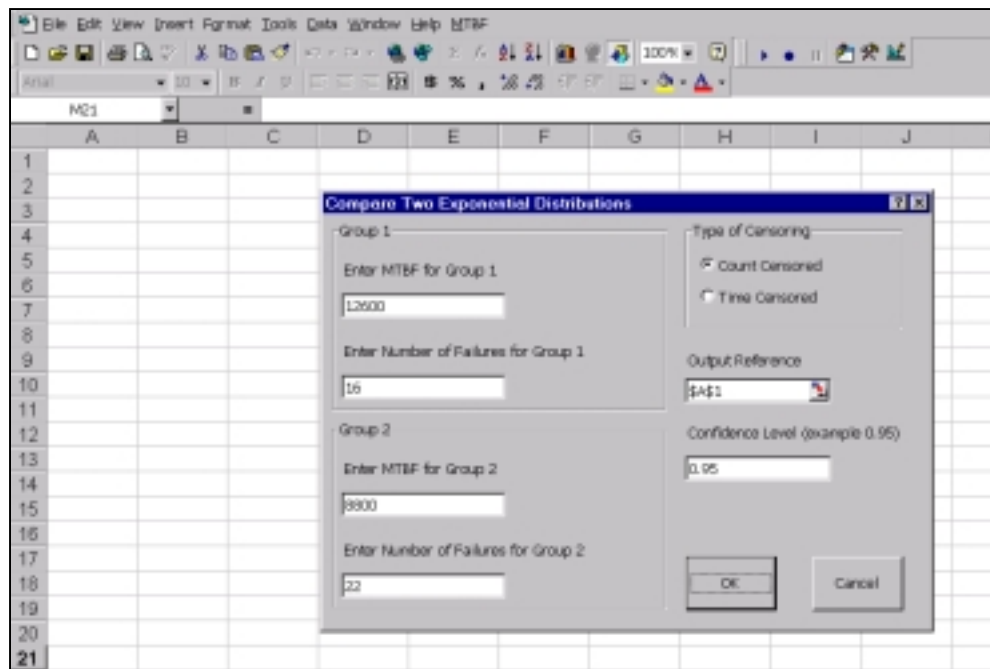
UCL 5.10

Interval excludes 1.

Significantly different.

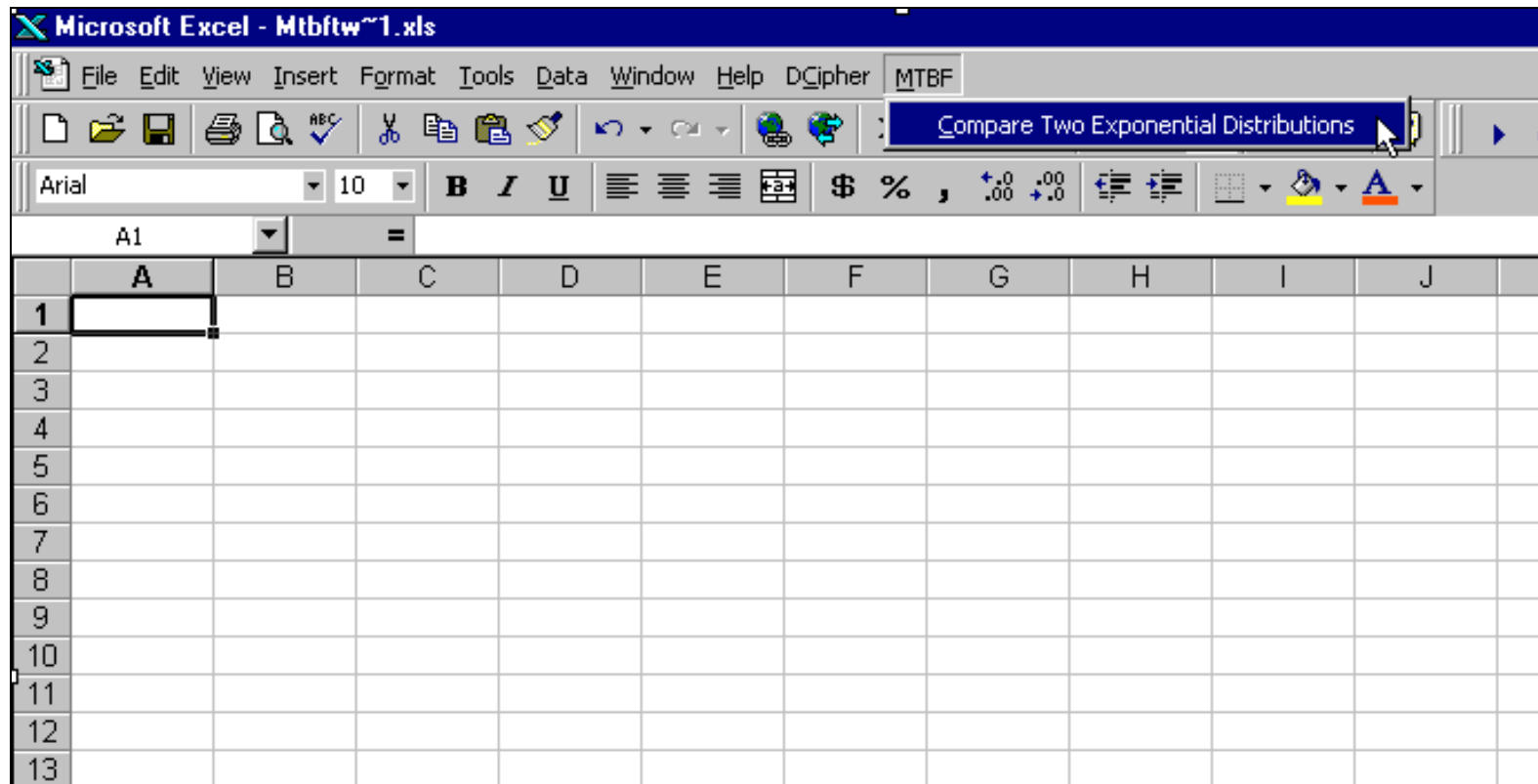
EXCEL Add-In

- This visual basic add-in makes the comparison of exponential distributions a snap!



Software Example

Simply Execute Our User-friendly Macro By Using the Pull Down Menu Provided.



Software Example Cont.

- **Select the Censoring Type.**
- **Fill In The MTBF and Number of Failures for Each Distribution, as shown below.**

Compare Two Exponential Distributions

Group 1

Enter MTBF for Group 1
12600

Enter Number of Failures for Group 1
16

Group 2

Enter MTBF for Group 2
8800

Enter Number of Failures for Group 2
22

Type of Censoring

Count Censored
 Time Censored

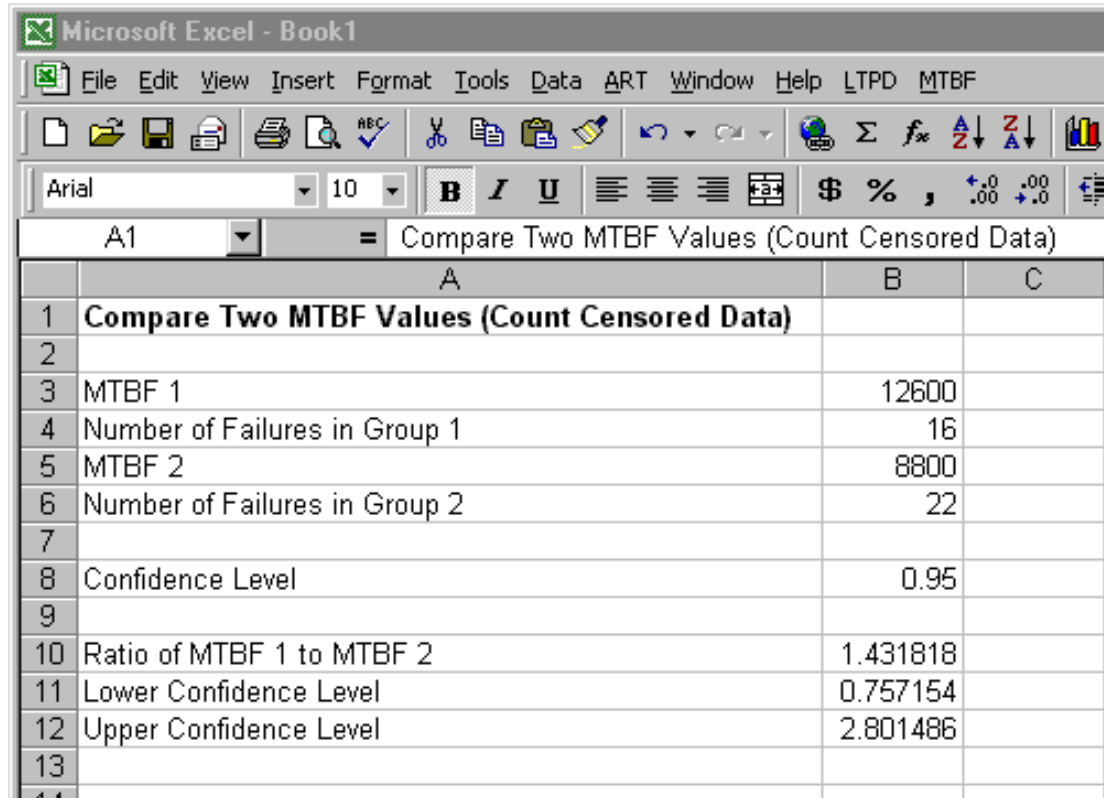
Output Reference
\$A\$1

Confidence Level (example 0.95)
0.95

OK Cancel

Software Example Cont.

**Click the “OK” Button, and Let the Software Do the Rest.
We Do All of the Calculations For You!**



The screenshot shows a Microsoft Excel spreadsheet with the following data:

	A	B	C
1	Compare Two MTBF Values (Count Censored Data)		
2			
3	MTBF 1	12600	
4	Number of Failures in Group 1	16	
5	MTBF 2	8800	
6	Number of Failures in Group 2	22	
7			
8	Confidence Level	0.95	
9			
10	Ratio of MTBF 1 to MTBF 2	1.431818	
11	Lower Confidence Level	0.757154	
12	Upper Confidence Level	2.801486	
13			
14			

Summary

- Simple procedures provided to compare two exponential distributions
- Excel add-in availability at
www.trindade.com/mttf.htm

References

- Bain, L.J. and Englehardt, M. (1991) *Statistical Analysis of Reliability and Life-Testing Models*, 2nd Ed., Marcel Dekker, New York
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